Stability of a cosmological model with dynamical cancellation of vacuum energy

A.D. Dolgov[(a)(b)(c)] and **M. Kawasaki**[(c)]

[(a)]*INFN, sezione di Ferrara, Via Paradiso, 12 - 44100 Ferrara, Italy*
[(b)]*ITEP, Bol. Cheremushkinskaya 25, Moscow 113259, Russia.*
[(c)]*Research Center for the Early Universe, Graduate School of Science,*
University of Tokyo, Tokyo 113-0033, Japan

Abstract

The stability of cosmological solutions in the recently suggested specific mechanism of dynamical compensation of vacuum energy is studied. It is found that the solutions in the original version lead to cosmological singularity which could be reached in final (and short) time. A modification of the interaction of the compensating field with gravity is suggested which allows to escape such singularity. It is shown that generic cosmological solution in this model tends to the Friedmann expansion regime even starting from initially large vacuum energy.

In a recent paper [1] we considered a mechanism of gross reduction of vacuum energy, ρ_{vac}, by a scalar field, ϕ, coupled to gravity in non-minimal and rather unusual way (see below Eq. (1)). The energy of the condensate of this field in de Sitter background could diminish ρ_{vac} down to the cosmological critical energy density, $\rho_c(t) \sim m_{Pl}^2/t^2$, transforming exponential cosmological expansion into the usual Friedmann one. The resulting equation of state becomes different from the vacuum one, $p = -\rho$. The idea of dynamical adjustment of vacuum energy is quite old [2] but no convincing and realistic mechanism had been found for a long time. Though lacking a realistic model, the mechanism of adjustment has two impressive and attractive features: first, the compensation of vacuum energy with the desired accuracy of about 100 orders of magnitude and, second, not complete compensation but only down to time dependent remnant, $\sim m_{Pl}^2/t^2$. The latter was in fact a generic prediction of adjustment models [2, 3] long before cosmological dark energy had been discovered [4]. For a review of the problem of vacuum energy, see e.g. Refs. [5].

However, the concrete models of adjustment discussed in the literature [2, 3, 6, 7] suffer from numerous shortcomings and realistic cosmology which includes dynamical adjustment of vacuum energy has yet to be found. It seems clear that vacuum and dark energies are surely related and without solution of the problem of compensation of vacuum energy any model of dark energy cannot be considered as complete, though phenomenological suggestions [8] can be quite useful.

In our paper [1] we considered a modification of the model of Ref. [7] and as a result we were able to obtain realistic cosmological solutions with dynamical compensation of vacuum energy and non-compensated remnants or, better to say, an excessive energy density of ϕ being cosmological dark energy. To our mind the mechanism described in [1] can be considered as moderately successful in the sense that such solutions indeed exist. However, their stability has not been studied and its suspected absence may present a serious challenge to a possibility of simultaneous solution of both vacuum and dark energy problems in the considered frameworks. In this paper we study the stability of the solutions presented in

Ref. [1] and find that they are unstable with respect to small perturbations. However, with some modification of the underlying Lagrangian governing interaction of ϕ with gravity stability can be achieved but at the expense of a realistic transition from radiation domination stage to matter one.

The action of the considered model has the form:

$$A = \int d^4x \sqrt{g} \left[-\frac{(R + 2\Lambda)}{2} + \frac{D_\mu \phi D^\mu \phi}{2R^2} - U(\phi) \right] \tag{1}$$

where the metric has the signature $(+, -, -, -)$, D_μ is the covariant derivative in this metric, and $g = -\det[g_{\mu\nu}]$. We took the units such that $m_{Pl}^2/8\pi = 1$. Correspondingly the cosmological constant Λ is expressed through the vacuum energy as $\rho_{vac} = \Lambda m_{Pl}^2/8\pi = \Lambda$. The explicit form of the potential $U(\phi)$ is not essential because solutions tend to $\phi = \phi_0 = const$ and only the magnitude of the derivative of the potential at this point, $U'(\phi_0)$, determines asymptotic behavior of the solution. Hereafter, we take $\phi_0 = 0$ for simplicity.

In the cosmological Friedmann-Robertson-Walker (FRW) background equation of motion for spatially homogeneous field $\phi = \phi(t)$ takes the form:

$$\left(\frac{d}{dt} + 3H \right) \left(\frac{\dot{\phi}}{R^2} \right) + U'(\phi) = 0. \tag{2}$$

while the Einstein equations acquire additional terms related to ϕ:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{D_\mu \phi D_\nu \phi}{R^2} + \frac{(D_\alpha \phi)^2}{2R^2} \left(g_{\mu\nu} + \frac{4R_{\mu\nu}}{R} \right)$$
$$-g_{\mu\nu} \left[U(\phi) + \rho_{vac} \right] + 2 \left(g_{\mu\nu} D^2 - D_\mu D_\nu \right) \frac{(D_\alpha \phi)^2}{R^3} = T_{\mu\nu}, \tag{3}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of the usual matter and $(D\phi)^2 \equiv D_\alpha \phi D^\alpha \phi$.

Taking trace over $(\mu - \nu)$ we obtain equation governing evolution of the curvature scalar R:

$$R - 3 \frac{(D\phi)^2}{R^2} + 4 \left[U(\phi) + \rho_{vac} \right] - 6D^2 \frac{(D_\alpha \phi)^2}{R^3} = T \tag{4}$$

where $T = T_\mu^\mu$. For the spatially homogeneous case the covariant D'Alambertian is $D^2 = d^2/dt^2 + 3Hd/dt$ and the Hubble parameter, H, is expressed through R as

$$R = -6 \left(2H^2 + \dot{H} \right) \tag{5}$$

As is found in Ref. [1], these equations allow power law solutions, $H = h/t$, $R = r/t^2$, and $[U(\phi) - \rho_{vac}] \sim 1/t^2$. An interesting feature of this solution is that the term containing D^2 in Eq. (4) is subdominant and can be neglected. However, for the study of stability one, as is well known, must retain higher derivative terms in differential equation of motion. We checked numerically that the solution of equations (2,4,5) quickly becomes singular with H and/or R going to infinity in finite time. General solution leads to a change of sign of R from initially negative value to a positive one and to subsequent collapse ($H \to -\infty$) as shown in Fig. 1 for $\rho_m = 0$ and Fig. 2 for $\rho_m \neq 0$. On the other hand, when the potential U is flatter, i.e. smaller U', the scalar field first changes sign and then all variable (ϕ, H and R) blow up for $\rho_m = 0$ (Fig. 3).[1] In this way a catastrophic cosmological singularity is quickly approached.
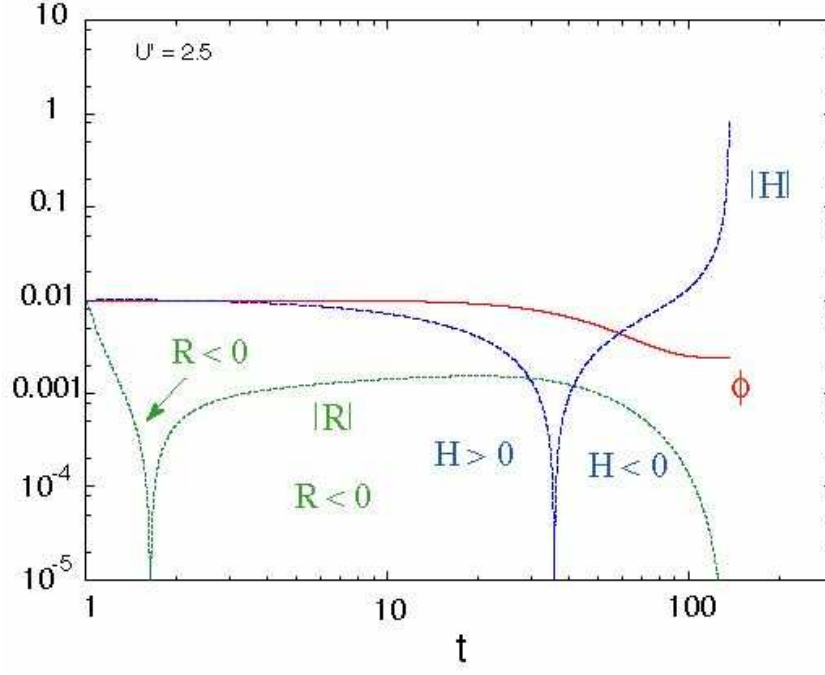
2

Figure 1: Evolution of the scalar ϕ, Hubble H and the curvature R for the potential $U' = 2.5$ and $\rho_m = 0$. For initial conditions we take $\phi(0) = 0.01$, $H(0) = 0.01$, $R(0) = -0.01$, $\phi'(0) = 10^{-5}$ and $R'(0) = 0$.
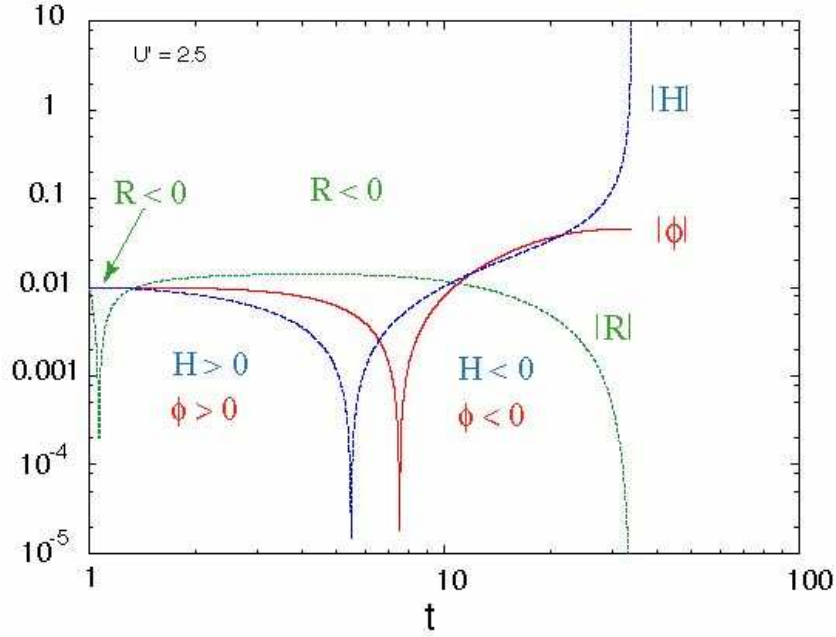

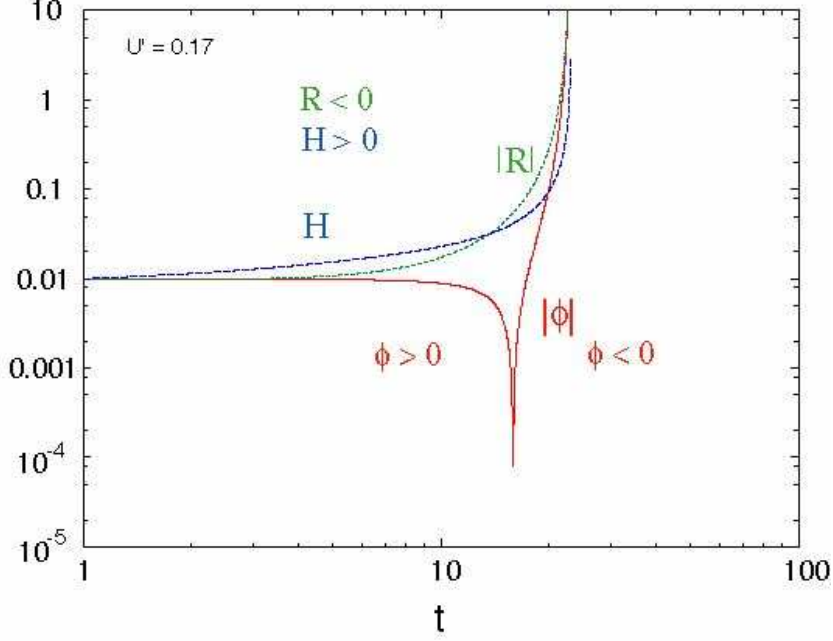Figure 2: Same as Fig. 1 except for $\rho_m(0) = 1$.

Figure 3: Same as Fig. 1 except for $U' = 0.17$.

To cure this unpleasant behavior we changed the Lagrangian (1) in such a way that the solution of equations of motion for ϕ (2), R (4) and H (5) does not lead to the catastrophic collapse of the universe for any sign of the curvature R. This may be done if the Lagrangian is allowed to depend upon the absolute value of R.[2] If we modify the kinetic term of the ϕ-field as

$$\frac{(D\phi)^2}{R^2} \to -\frac{(D\phi)^2}{R\,|R|} \tag{6}$$

then $|H|$ would not infinitely rise for any sign of R. Such non-analytical terms in the Lagrangian are rather unusual but may be allowed for a toy model. One may have slightly better form for non-analytic terms as e.g. $\sqrt{R^2 + (D_\alpha\phi)^2}$. To avoid negative sign of the expression under square root the solution should ensure $R^2 > |(D\phi)^2|$, or if it is not so the expression could be properly modified e.g. by changing $(D\phi)^2$ into $(D\phi)^4$.

It is convenient to introduce additional notations:

$$z = \frac{\dot{\phi}}{R\,|R|} \text{ and } w = \frac{(\dot{\phi})^2}{|R|^3} \tag{7}$$

In terms of these new functions and in spatially homogeneous case we obtain the following system of equations governing cosmological evolution in the model under consideration:

$$\dot{\phi} - \frac{|w|\,w}{z^3} = 0, \tag{8}$$

$$\dot{z} + 3Hz - U'(\phi) = 0, \tag{9}$$

---

[1]If there exists significant ρ_m the solution behaves as seen in Fig. 2 even for small U'.

[2]This modification cannot cure the blow up of ϕ, H and R in the case of small U' and ρ_m since this instability arises without change of the curvature sign.

$$\dot{H} + 2H^2 + R/6 = 0, \tag{10}$$

$$6D^2w + 3\frac{w^2}{z^2} + \frac{|w|}{z^2} + 4\frac{|w|}{w}\left[U(\phi) + \rho_{vac}\right] + \frac{|w|}{w}\,T = 0. \tag{11}$$

The evolution of the trace of the energy-momentum tensor of the usual matter is determined by its equation of state. For relativistic matter $T = 0$, while for non-relativistic matter:

$$\dot{T} = 3HT \tag{12}$$

The system of equations (8-12) has been solved numerically, see figs. 4 and 5. We have found, for more or less general initial conditions, that the curvature, R, oscillates around zero with an increasing frequency and with the amplitude decreasing somewhat faster than $1/t^2$. Possibly particle production by quickly oscillating curvature would make the magnitude of R to decay even faster. The Hubble parameter tends to $H = 1/(2t)$, as is the case of cosmology dominated by relativistic matter. The compensating scalar field ϕ tends to a constant value oscillating around the latter. In the case that non-relativistic matter is absent and thus, $T = 0$ the model corresponds to reasonable Friedmann cosmology with the usual expansion law, $a(t) \sim t^{1/2}$. However, if non-relativistic matter is present and T is non-zero it would start to dominate the total matter energy density, $T = \rho_m \sim a^{-3}$, while the expansion regime remains relativistic, $a \sim t^{1/2}$.

Still the solutions found in Ref. [1] also exist but to "hit" them a fine-tuning of initial conditions is necessary. Moreover, these solutions are unstable and small fluctuations around them have one (out of 5) rising mode. It is interesting that if one neglects D^2-term in Eq. (4), as we have done in Ref. [1], the "good" solutions found there are stable but the transition from relativistic to non-relativistic regime encounters instability.

So only a moderate success can be reported. The model presented above allows for the transformation of de Sitter cosmological solution, driven by vacuum energy, into the Friedmann one but the expansion regime is always relativistic with $H = 1/(2t)$ independently of the matter content. The matter dominated (MD) solution with $H = 2/(3t)$ is also possible but such solution is unstable, small fluctuations around it would drive the system back to radiation dominated (RD) regime. Moreover, starting from RD regime, assuming that T is small, we have not found any solution which allows for the change to MD regime when T begins to dominate matter density. Despite these shortcomings, one may hope that the resolution of these problems may be feasible with some modification of the Lagrangian describing interaction of the compensating field with gravity. Maybe such constructions resembles epicycles in Ptolemaic astronomy but there surely was some truth in the latter. Anyhow, adjustment mechanism seems to be the only one that may possibly solve simultaneously both problems of 100 orders of magnitude compensation of vacuum energy and to explain the non-compensated remnant in cosmological energy density (dark energy) which contributes a relative fraction of order of unity into cosmological energy density at any time of the universe history.
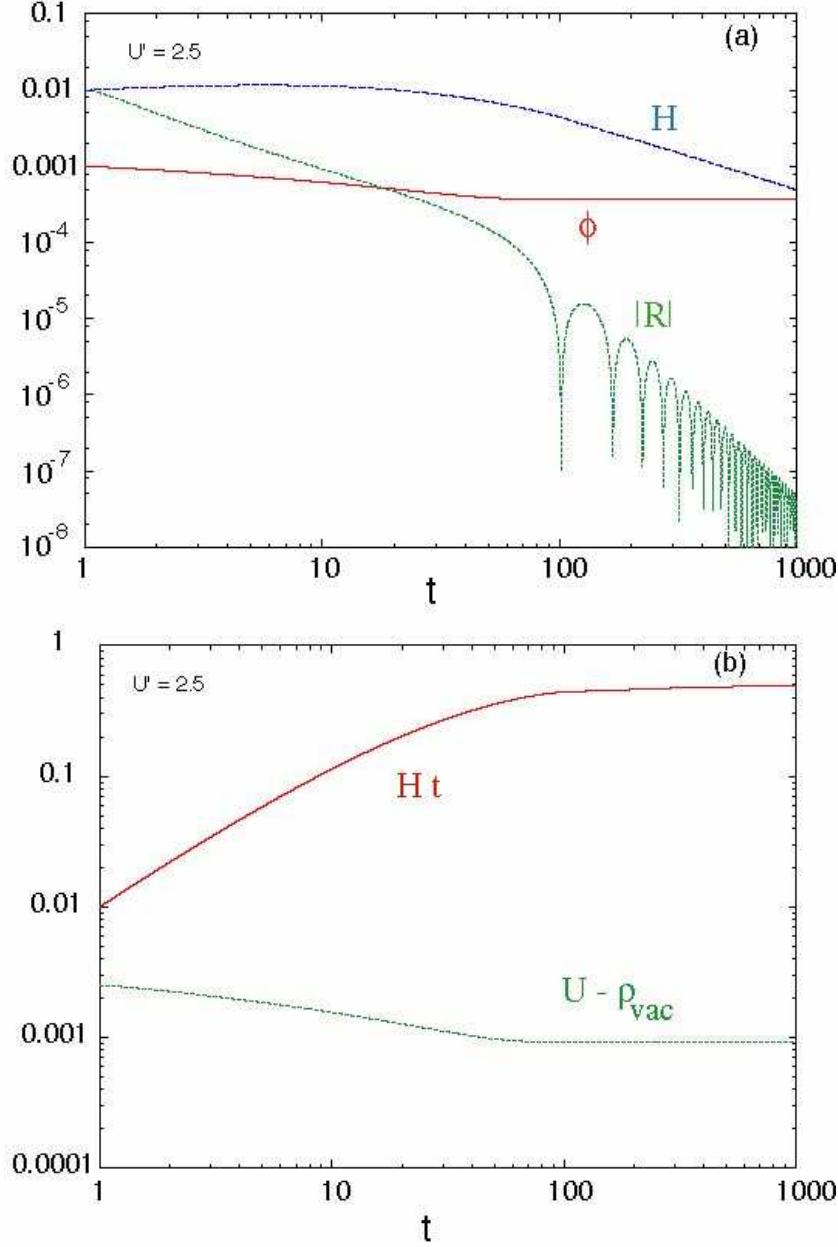
Figure 4: (a) Evolution of the scalar ϕ, Hubble h and the curvature R for the modified kinetic term Eq.(6). We take $U' = 2.5$ and $\rho_m = 0$. The initial values are $\phi(0) = 0.001$, $H(0) = 0.01$, $R(0) = -0.01$, $\phi'(0) = 10^{-4}$ and $R'(0) = 0$. (b) Evolution of $h = Ht$ and $U - \rho_{vac}$.

References

[1] A.D. Dolgov, M. Kawasaki, astro-ph/0307442.

[2] A.D. Dolgov, In "The very early universe." Eds. G.Gibbons, S.W.Hawking, and S.T.Tiklos, Cambridge University Press, 1982, p. 449.
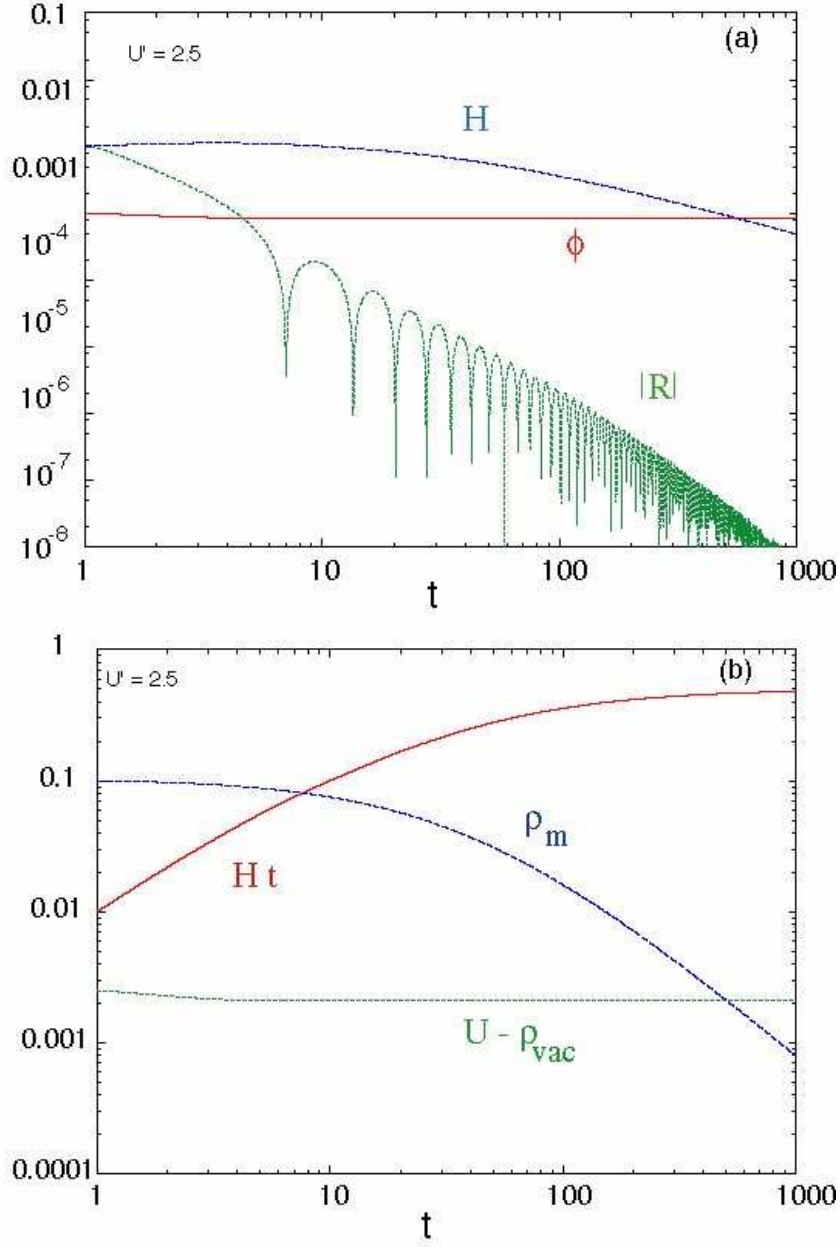
Figure 5: Same as Fig. 4 except for $\rho_m(0) = 0.1$.

[3] A.D. Dolgov, ZhETF Letters, 41 (1985) 280;
    A.D. Dolgov, Phys. Rev. D55 (1997) 5881.

[4] A.G. Riess *et al.* [Supernova Search Team Collaboration], Astron. J. **116**, 1009 (1998);
    S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Astrophys. J. **517**,
    565 (1999);
    J.L. Tonry *et al.*, astro-ph/0305008;
    A.T. Lee et al, Astrophys. J. **561**, L1 (2001);
    C.B. Netterfield et al, Astrophys. J. **571** (2002) 604;

N.W. Halverson et al, Astrophys. J. **568** (2002) 38;
D.N. Spergel et al, astro-ph/0302209.

[5] S. Weinberg, Rev. Mod. Phys. 61 (1989) 1;
A.D. Dolgov, Proc. of the XXIVth Rencontre de Moriond; Series: Moriond Astrophysics Meetings, Les Arcs, France, p.227 (1989); ed. J. Adouse and J. Tran Thanh Van;
A.D. Dolgov, astro-ph/9708045, Talk given at the 4th Paris Cosmology Colloquium, Paris, France, 4-6 Jun 1997. In Paris 1997, Phase transitions in cosmology, p. 161;
P. Binetruy, Int. J. Theor. Phys. 39 (2000) 1859 (lectures at Les Houches summer school 'The early Universe" and Peyresq 4 meeting, July 1999);
V. Sahni, A. Starobinsky, Int. J. Mod. Phys. D9, 373 (2000);
A.D. Dolgov, hep-ph/0203245, Contribution to M. Novello Festschrift;
N. Straumann, astro-ph/0203330, Invited lecture at the first Séminaire Poincaré, Paris, March 2002;
T. Padmanabhan, Phys. Rept. 380 (2003) 235;
J. Yokoyama, gr-qc/0305068, Proc. JGRG12 (12th workshop on general relativity and gravitation in Japan).

[6] R.D. Peccei, J. Sola, and C. Wetterich, Phys. Lett. B195 (1987) 18;
L.H. Ford, Phys. Rev. D35 (1987) 2339;
S.M. Barr and D. Hochberg, Phys. Lett. B211 (1988) 49;
Y. Fujii and T. Nishioka, Phys. Rev. D42 (1990) 361; Phys.Lett. B254 (1991) 347 and references therein;
A.D. Dolgov, Yukawa Int. Preprint YITP/K-924 (1991) (unpublished);
V.A. Rubakov, Phys.Rev. D61 (2000) 061501;
A. Hebecker, C. Wetterich, Phys. Rev. Lett. 85 (2000) 3339.

[7] S. Mukohyama, L. Randall, hep-th/0306108.

[8] R.R Caldwell, R. Davis, P.J. Steinhardt, Phys. Rev. Lett. 80 (1998) 1582.